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13. ABSTRACT (Maximum 200 words) <p>Two separate investigations have been made: (1) evaluation of various forms of Burnett equations for 2D flows including the frame dependent as well as frame independent forms; and (2) exploration of the compatibility of Burnett equations with the second law of thermodynamics.</p> <p>Among eight different Burnett formulations investigated, six were found either to be impractical for 2D/3D computations, or to yield unphysical heat conduction. The remaining two forms yielded virtually identical numerical results, which led to an investigation that has revealed their previously unsuspected analytical identity. The frame dependence problem for Burnett equations is therefore not fully resolved.</p> <p>Detailed analysis of Burnett forms for the rate of irreversible production of entropy has demonstrated incompatibility with the second law of thermodynamics for certain flows.</p> <p style="text-align: right;">DTIC QUALITY INSPECTED 3</p>				
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# INVESTIGATION OF BURNETT EQUATIONS FOR TWO-DIMENSIONAL HYPERSONIC FLOW

Research from December 1, 1993 to November 30, 1994

AFOSR Contract F49620-94-1-0055

## Abstract

The subject research comprises two separate areas of investigation with the broad objective of exploring results from numerical computations of two-dimensional flow fields using Burnett equations and, by way of comparison, also Navier-Stokes equations. Unlike the latter set of motion equations which have remained unchanged for a century and half, a number of different forms of Burnett equations have been advanced subsequent to the original form derived by D. Burnett in 1935.

One objective of our research program, therefore, has included an evaluation of each of these differing forms. This evaluation was carried out first for one-dimensional shock wave flows as described in our *Final Report* on AFOSR contract 92J0012 ending October 31, 1993, and in a publication referred to in that report (Welder et al., 1993). A corresponding evaluation for two-dimensional flows is reported on herein.

A second initial objective was to investigate, using both Navier-Stokes and Burnett equations, the oblique shock on cowl lip interaction at high altitudes, a phenomenon relevant to hypersonic scram jet inlets. This airframe-inlet interaction is strongly altitude dependent, and our results using Navier-Stokes equations (Comeaux, 1993) were also described in the final report on contract 92J0012 cited above.

During this past year, shortly after beginning investigation of Burnett solutions for the 2D shock interaction flow, it was found that the Burnett formulation could be of questionable compatibility with the second law of thermodynamics. Consequently, a thorough investigation was undertaken of the irreversible production of entropy as characterized by Burnett equations, employing both a thermodynamic analysis and a kinetic theory analysis. This

turned into a fundamental discovery about Burnett equations, as outlined subsequently in the present report, and in more detail in an attachment hereto. Our contract terminated before solutions to Burnett equations could be obtained for the shock on cowl lip interaction phenomena. The last publication on the research (Comeaux et al., 1995) has been prepared for, and will be presented at, the January 1995 AIAA meeting in Reno, Nevada. An advance preprint is appended hereto as part of our final report.

### Evaluation of Various Forms of Burnett Equations

In the historical development of Burnett type equations, which are of higher order than the familiar Navier-Stokes equations, five different formulations have been advanced thus far, as follows:

Formulation	Author/year	Accession For	
		NTIS CRA&I DTIC TAB Unannounced Justification	<input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
Original Burnett	Burnett 1935	By _____	
Conventional Burnett	Chapman-Cowling 1939	Distribution /	
Frame Indifferent Burnett	Woods 1983	Availability Codes	
Augmented Conventional Burnett	Zhong et al. 1991	Dist	Avail and / or Special
Woods	Woods 1992	A-1	

The "original" Burnett formulation contains a material derivative of velocity in the viscous-stress tensor, and a material derivative of temperature in the heat-flux vector. Chapman and Cowling replaced these material derivatives by spacial gradient terms, using approximate forms of the momentum and energy conservation equations, to obtain what we term the "conventional" Burnett equation. Since these forms were found in the 1970's to be frame dependent in a rotating coordinate system, Woods attempted to formulate a "frame indifferent" version of Burnett order equations retaining explicitly without approximation

the material derivatives in viscous stress and heat flux. Following discovery in 1982 that the conventional Burnett equations were unstable to small-disturbance sound waves of wave length less than about one mean free path, Zhong added a few select terms of higher than Burnett order to stress and heat flux in order to stabilize the equation set, and thereby constructed the "augmented conventional" Burnett equations. Finally, using a derivation method completely different from the kinetic theory of Chapman-Enskog-Burnett, Woods formulated in 1992 an alternate set of equations which differed considerably from previous formulations. Since two of the five main forms above can be modified by using an approximation for the material derivatives more accurate (and complex) than the approximation used by Chapman and Cowling, and since Woods has added a separate truncated version to his 1992 formulation of Burnett equations, eight different formulations altogether have been investigated in the present research project.

A note is in order about the process of replacing material derivatives by spacial gradients. Chapman and Cowling used the Euler form of momentum and energy conservation to replace the material derivatives by spacial gradients given for the  $x$ -direction gradients by

$$\begin{aligned}\frac{D}{Dt}\left(\frac{\partial u}{\partial x}\right) &= -\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\right)^2 \\ \frac{D}{Dt}\left(\frac{\partial T}{\partial x}\right) &= -\frac{\partial}{\partial x}\left(\frac{p}{\rho c_v}\frac{\partial u}{\partial x}\right) - \frac{\partial u}{\partial x}\frac{\partial T}{\partial x}\end{aligned}\tag{1}$$

This approximation leads to a Burnett equations set correct to Burnett order, although it differs from the exact representation of material derivatives given for the  $x$ -direction gradients by

$$\begin{aligned}\frac{D}{Dt}\left(\frac{\partial u}{\partial x}\right) &= \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) \\ \frac{D}{Dt}\left(\frac{\partial T}{\partial x}\right) &= \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial x}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\end{aligned}\tag{2}$$

An approximation more accurate though more complex than (1) is given by the Navier-Stokes form of momentum and energy conservation

$$\begin{aligned}
\frac{D}{Dt}\left(\frac{\partial u}{\partial x}\right) &= \frac{\partial}{\partial x}\left(-\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{4}{3}\frac{1}{\rho}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)\right) - \left(\frac{\partial u}{\partial x}\right)^2 \\
\frac{D}{Dt}\left(\frac{\partial T}{\partial x}\right) &= \frac{\partial}{\partial x}\left[\frac{1}{\rho c_v}\left(-p\frac{\partial u}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2\right)\right] - \frac{\partial u}{\partial x}\frac{\partial T}{\partial x}
\end{aligned} \tag{3}$$

which is the basis for two of the alternate forms of Burnett equations evaluated in this research.

Whereas the alternate forms using the Navier-Stokes approximation for material derivatives were feasible to use in computations of 1D shock structure, as described by Welder et al. (1993), these forms were found not to be practical for use in 2D computations because of the very large increase in computer power required. Derivatives of (3) appear in the motion equations, which accumulate in large number, even for 2D flow, requiring 130 additional terms in stress, and 62 additional terms in heat conduction to be differenced and computed, as outlined in Table 1. Hence the Burnett variations using the Navier-Stokes approximation for material derivatives is not viewed as computationally practical at the present time, and is not considered further herein.

As shown in Welder et al. there is a fundamental difficulty even in 1D flows with Burnett formulations that do not make any approximation for material derivatives. Such formulations are the "exact" or original Burnett equations, and the "exact" Woods equations. These lead to unphysical heat conduction in the upstream portion of hypersonic shock-wave structure. By "unphysical" is meant heat energy being conducted from colder to hotter regions. This, as is demonstrated later, corresponds the irreversible subtraction rather than an irreversible production of entropy, conditions in violation of the second law of thermodynamics. The net result is that, with these various considerations taken into account, only four formulations are not excluded for further consideration, as indicated in Table 2.

Computations have been made of the 2D hypersonic flow over a flat plate at zero incidence using the four different surviving formulations of Burnett equations. Computed results for the case  $M_\infty = 11$ ,  $T_w = T_\infty = 300^\circ\text{K}$ , and argon gas are shown in Figures 1, 2, and 3. At the lowest Knudsen number of 0.02 (Figure 1), there is little difference between the four different Burnett formulations. In the legend of this and subsequent figures

"conventional Burnett" refers to the augmented conventional Burnett equations of Zhong, the augmented terms, being necessary for stable numerical computations. At  $Kn = 0.1$  (Figure 2) some differences appear and are further amplified at the highest computed  $Kn$  of 0.2 (Figure 3). This comparison indicates that the augmented conventional Burnett equations—labeled "conventional Burnett" in the figures—is the preferred form to use. The overall results are summarized in Table 3.

An unexpected result emerged from these computations. No appreciable difference in results were obtained between the conventional Burnett equations, demonstrated in the 1970's to yield frame dependent results in a rotating coordinate system, and the so-called "frame indifferent" Burnett equations of Woods. This unanticipated result lead to an analytical examination of the two formulations. It was discovered that, although they appear quite different in their vector-tensor form, certain vector-tensor operational identities can be employed to show that they are, in fact, identical. Hence, if the conventional Burnett equations are frame dependent, so then are the so-called "frame indifferent" equations of Woods. The issue of frame dependence or indifference, therefore, is clearly not yet fully resolved.

### **Burnett Equations and the Second Law of Thermodynamics**

Investigation of this particular subject was in no way anticipated when the reserach project started. Yet it has yielded, in our view, by far the most scientifically significant result of the overall research project. In short, it has been found by our research assistant Keith Comeaux, from a very thorough development of the entropy production equation for Burnett equations, that these equations unfortunately do not always conform to the second law of thermodynamics. A paper recently has been prepared on the subject for presentation at the January 1995 annual meeting of the AIAA in Reno. An advance reprint entitled "An Analysis of the Burnett Equations Based on the Second Law of Thermodynamics", by Comeaux, K. A., Chapman, D. R., and MacCormack, R. W., *AIAA Paper No. 95-0415*, is

appended hereto for details of the analysis.

In this paper, two entirely independent derivations are developed for the rate of irreversible entropy production embodied in Burnett equations. The first method of derivation is based solely on thermodynamics, including the Clausius-Duhem inequality and the Gibbs equation, and leads to the complex equation (2.20) for Burnett entropy production in the appended reprint. The second method of derivation is based solely on the Chapman-Enskog kinetic theory, including the Boltzman equation, H-theorem, and statistical mechanics definition of entropy. It is shown that this physically independent derivation leads to precisely the same complex equation for entropy production.

Physical interpretations are given in the appended paper for the various terms in the Burnett entropy production equation to clarify their meaning. Some example flows are then treated showing that in certain portions of some Burnett flows the second law of thermodynamics is violated. One such example occurs in the upstream portion of hypersonic shock structure. This is consistent with the unphysical heat conduction earlier noted in this same region by Welder et al. (1993). A second example occurs in sound waves of wave length less than the order of a mean free path. This is consistent with the computed non-physical amplification, rather than dissipation, of such short wave-length waves. In general, the violation of the second law appears to be responsible for a number of the computational instabilities that have been encountered in the past with the Burnett equations. It is our view that a completely new look at Burnett order equations, and possible restructuring, should be undertaken giving full attention to the requirement that such equation sets be fully compatible in all cases with the second law, just as the Navier-Stokes equations are in that manner compatible for all flows.

## References

- Welder, W. T., Chapman, D. R., and MacCormack, R. W. "Evaluation of Various Forms of the Burnett Equations," *AIAA Paper 93-3094*, 24th Fluid Dynamics Conference, July 6-9, 1993, Orlando, FL.
- Comeaux, K. A., Chapman, D. R., and MacCormack, R. W. "Viscous Hypersonic Shock-Shock Interaction on a Blunt Body at High Altitude," *AIAA Paper 93-2722*, 28th Thermophysics Conference, July 6-9, 1993, Orlando, FL.
- Comeaux, K. A., Chapman, D. R., and MacCormack, R. W. "An Analysis of the Burnett Equations Based on the Second Law of Thermodynamics," *AIAA Paper 95-0415*, to be presented at AIAA Annual Winter Meeting, January 1995, Reno, NV.



# N-S Based Approximation For Material Derivative

## Not Practical For 2D Flows

	1-D Terms		2-D Terms	
	Stress	Heat Cond.	Stress	Heat Cond.
Navier-Stokes	1	1	6	2
Conventional Burnett	6	3	42	22
Additional Navier-Stokes Based D/Dt Terms	6	7	130	62

Table 1

# State Of Burnett Type Expressions After 1-D Survey

	<u>Not Excluded</u>	<u>Excluded</u>	<u>Comments</u>
Exact Burnett		X	Incorrect Heat Conduction
Augmented Conventional Burnett	X		
N-S Based Burnett		X	Impractical for 2-D/3-D
Frame Indifferent Burnett	X		
Exact Woods		X	Incorrect Heat Conduction
Augmented Conventional Woods	X		
N-S Based Woods		X	Impractical for 2-D/3-D
Truncated Woods	X		

Table 2

# State Of Burnett Type Expressions After 1-D Shock And 2-D Blunt Body Survey

	<u>Not Excluded</u>	<u>Excluded</u>	<u>Comments</u>
Exact Burnett		X	Incorrect Heat Conduction
Augmented Conventional Burnett	X		
N-S Based Burnett		X	Impractical for 2-D/3-D
Frame Indifferent Burnett	X		
Exact Woods		X	Incorrect Heat Conduction
Augmented Conventional Woods		X	No Solution At Higher $Kn$
N-S Based Woods		X	Impractical for 2-D/3-D
Truncated Woods		X	Poor Comparison/No Solution At Higher $Kn$

Table 3

# Temperature Distribution Along Stagnation Streamline

$Kn = 0.02$

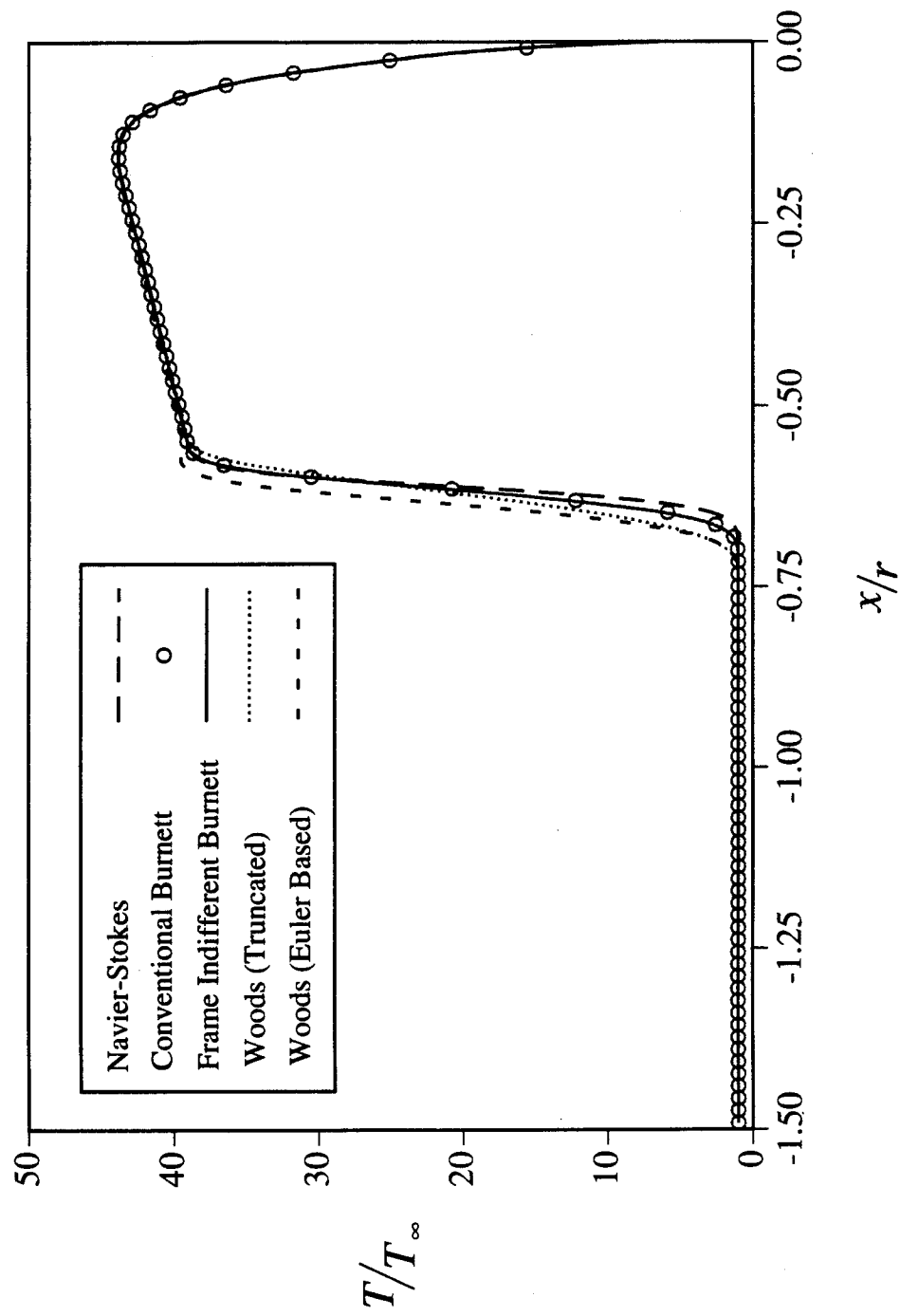


Figure 1

# Temperature Distribution Along Stagnation Streamline

$Kn = 0.1$

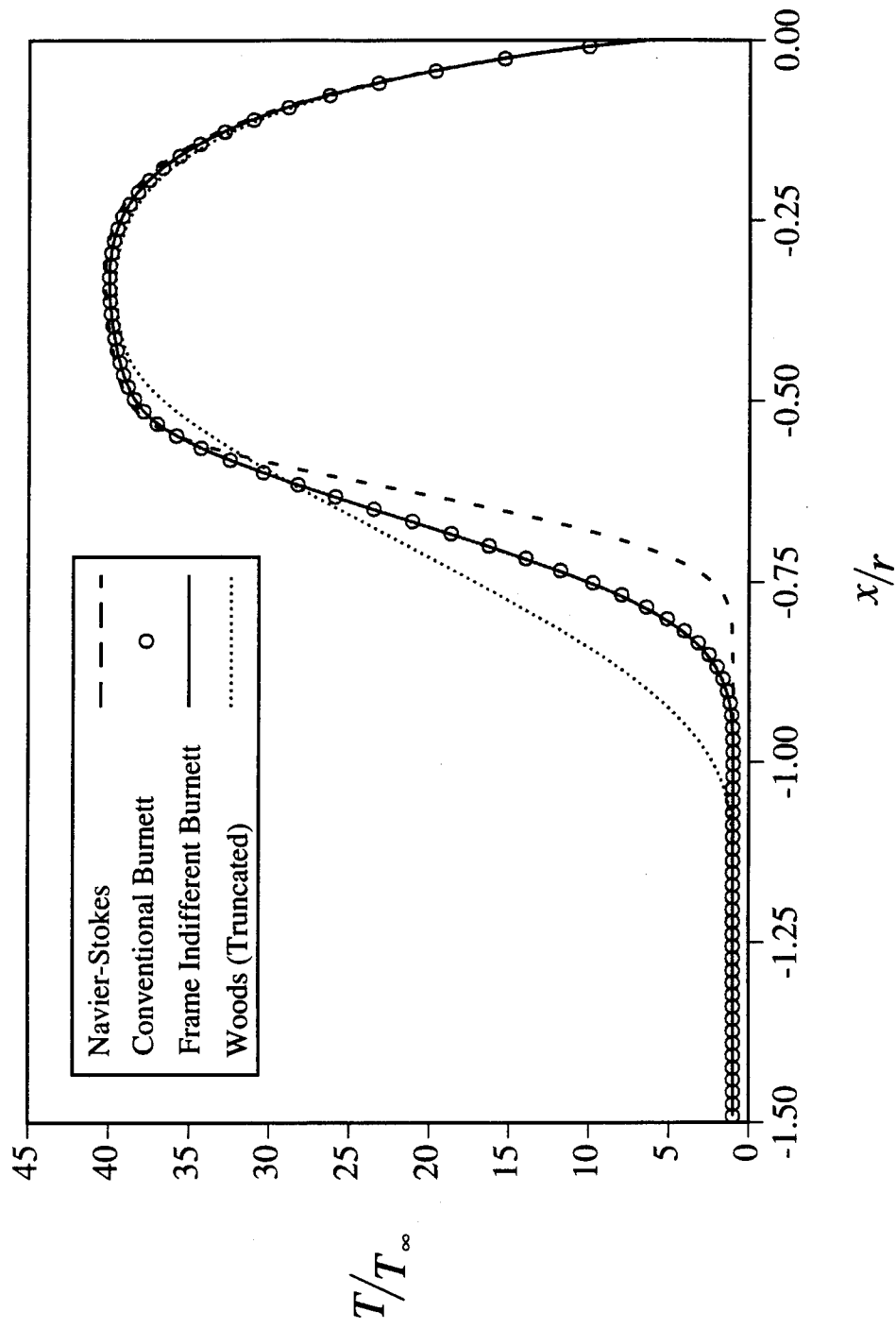


Figure 2

# Temperature Distribution Along Stagnation Streamline

$Kn = 0.2$

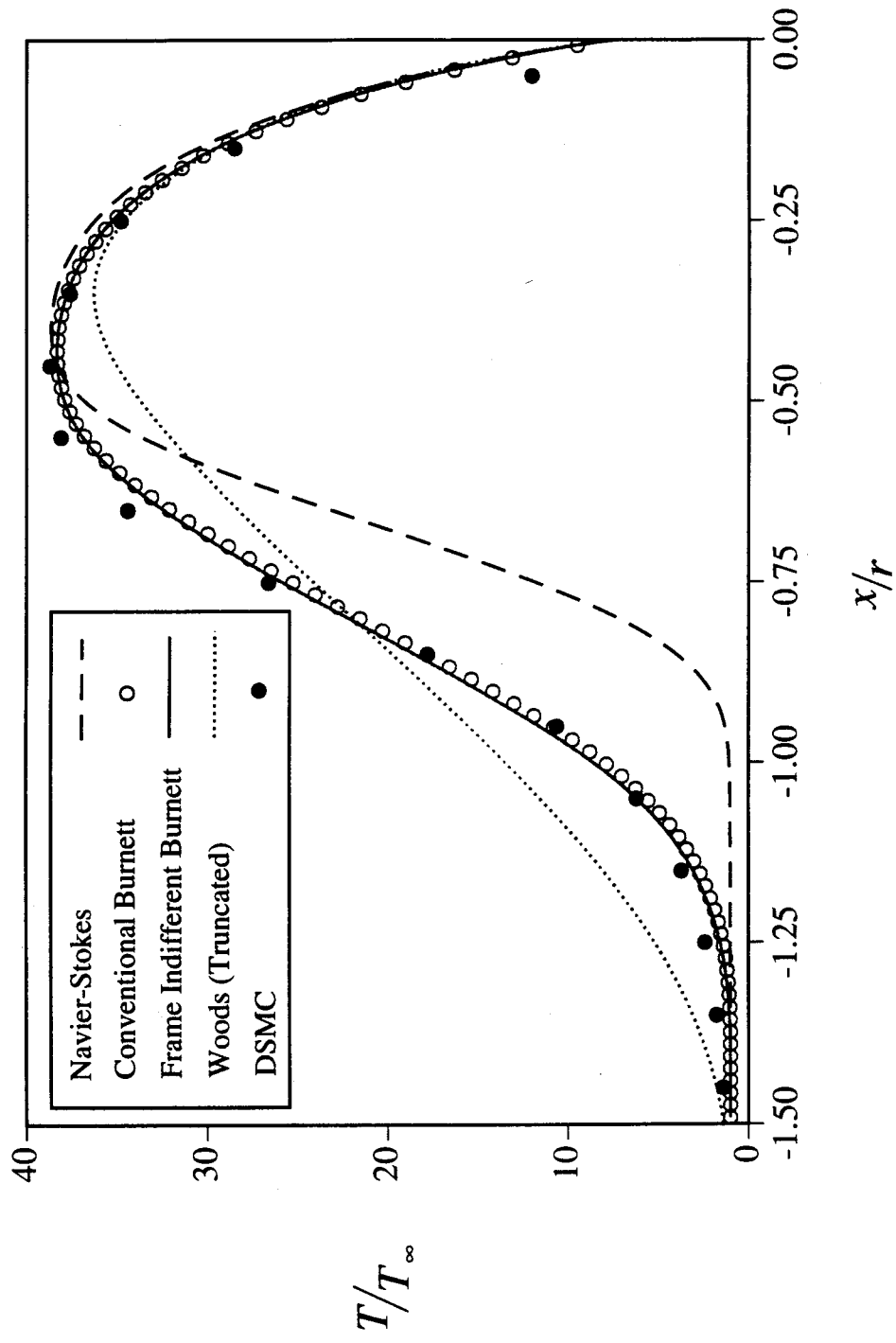


Figure 3